Vector Equation Error Adaptive IIR Filtering

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Abstract – We present a new equation error infinite impulse response (IIR) filtering algorithm using a vector equation error with block regressor. A new cost function, the squared norm of a posteriori vector equation error, to be minimized with respect to the changes of filter estimates is introduced. Consequently, we drive a new update equation with block regressor. Experimental results demonstrate that the proposed algorithm possesses the improved convergence performance.

1. Introduction

Adaptive IIR filtering methods are commonly classified into equation error (EE) and output error (OE) [1]. The EE approach for adaptive IIR filtering has such attractive features as a unimodal error surface, good convergence, and guaranteed system stability compared to the OE approach [1]. In spite of those advantages, the EE approach has not been widely used, since it may generate biased coefficient estimates in the presence of noise. Several algorithms are proposed to solve the bias problem [2]-[4].

Most existing EE IIR filterings have tried to minimize a squared scalar valued EE and use non-block regressor. However, it is known that the block regressor gives birth to the improved convergence performance of adaptive filter in case of a finite impulse response (FIR) filtering [5], [6]. For example, the convergence performances using vector EE with block regressor such as the affine projection algorithm [5] and data-reusing algorithms [6] are better than those with non-block regressor. Motivated by this fact, we incorporate EE IIR filtering with block regressors.

In this paper, we introduce a new EE IIR filtering algorithm using vector EE with block regressors, which we will call the vector equation error adaptive IIR filtering. A new cost function to minimize the squared norm of a posteriori error vector EE with respect to the changes of filter estimates is defined. By minimizing the new cost function, update equations with block regressors are derived. Experimental results show that the convergence performance of the proposed algorithm is better than that of conventional EE IIR filtering.

2. Vector Equation Error Adaptive IIR Filtering

2.1 EE IIR Filtering

Figure 1: Adaptive EE IIR filter in system identification configuration.

Let a zero-mean, discrete-time signal $x(i)$ be the input to an unknown system $H(z)$. Suppose that $H(z)$ is represented by a rational function

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{m=0}^{M} b_m z^{-m}}{\sum_{l=0}^{L} a_l z^{-l}},$$

where $a_l$ and $b_m$ are the adaptive filter coefficients and $l$ denotes a delay of $l$ samples. The output $y(i)$ of the unknown system $H(z)$ is represented by a difference equation such that

$$y(i) = d(i) - \left( \sum_{m=0}^{M} b_m x(i-m) - \sum_{l=0}^{L} a_l d(i-l) \right),$$

where it is assumed that $a_0 = 1$. The observed output $y(i)$ is a desired signal, corrupted by additive noise, in the system identification problem is given by

$$d(i) = y(i) + v(i),$$

where $v(i)$ is white measurement noise with variance $\sigma_v^2$.

Suppose that the sufficient order modeling situation is considered, $\hat{A}(z)$ and $\hat{B}(z)$ are given by

$$\hat{A}(z) = \sum_{l=0}^{L} \hat{a}_l z^{-l} \quad \text{and} \quad \hat{B}(z) = \sum_{m=0}^{M} \hat{b}_m z^{-m},$$

where $\hat{a}_l$ and $\hat{b}_m$ are the adaptive filter coefficient estimates $a_l$ and $b_m$, respectively. The EE, $e(i)$, is given by

$$e(i) = d(i) - \hat{A}(z) \hat{x}(i),$$

where $\hat{x}(i)$ is given by

$$\hat{x}(i) = \hat{A}(z) \hat{x}(i-1) - \hat{B}(z) \hat{y}(i).$$
\[ \hat{a}_i = [\hat{a}_i(0) \ \hat{a}_i(1) \ \ldots \ \hat{a}_i(i)], \]
\[ \hat{b}_i = [\hat{b}_i(0) \ \hat{b}_i(1) \ \ldots \ \hat{b}_i(i)], \]

where \( \begin{bmatrix} D_1^T D_1 + \epsilon_a I_{L+1} & D_1^T X_i \\
X_i^T D_1 & X_i^T X_i + \epsilon_e I_{M+1} \end{bmatrix} \) is positive definite and \( I_k \) is \( K \times K \) identity matrix. Rewriting (13) with respect to \( \Delta \hat{a}_{i+1} \) and \( \Delta \hat{b}_{i+1} \) yields
\[ \begin{bmatrix} \Delta \hat{a}_{i+1} \\
\Delta \hat{b}_{i+1} \end{bmatrix} = \begin{bmatrix} D_1^T D_1 + \epsilon_a I_{L+1} & D_1^T X_i \\
X_i^T D_1 & X_i^T X_i + \epsilon_e I_{M+1} \end{bmatrix}^{-1} \begin{bmatrix} D_1^T e_i \\
X_i^T e_i \end{bmatrix}. \tag{14} \]

By matrix inversion lemma [7],
\[ \begin{bmatrix} D_1^T e_i \\
X_i^T e_i \end{bmatrix} = \begin{bmatrix} e_a I_{L+1} & 0 \\
0 & e_e I_{M+1} \end{bmatrix}^{-1} \begin{bmatrix} D_1^T e_i \\
X_i^T e_i \end{bmatrix} \]
\[ = \begin{bmatrix} D_1^T e_i \\
X_i^T e_i \end{bmatrix} \tag{15} \]

Substituting (15) into (14) and using (9) and (10) produce the following update equations:
\[ \hat{a}_{i+1} = \hat{a}_{i+1} + \frac{\mu_a}{\epsilon_a} D_1^T \left( I_k + \frac{1}{\epsilon_a} D_1 D_1^T + \frac{1}{\epsilon_e} X_i X_i^T \right)^{-1} e_i, \tag{16} \]
\[ \hat{b}_{i+1} = \hat{b}_{i+1} + \frac{\mu_e}{\epsilon_e} D_1^T \left( I_k + \frac{1}{\epsilon_a} D_1 D_1^T + \frac{1}{\epsilon_e} X_i X_i^T \right)^{-1} e_i, \tag{17} \]

where \( \mu_a \) and \( \mu_e \) are the step sizes which control the convergence speed and the misadjustment error after convergence.

From (16) and (17), we know that \( \epsilon_a \) and \( \epsilon_e \) take part in the convergence performance with \( \mu_a \) and \( \mu_e \). In case of \( \epsilon_a = \epsilon_e \), a large regularization parameter will ensure a small effective step size and thus the proposed algorithm results in a small misadjustment error in steady state, but converges slowly. On the other hand, a small regularization parameter will yield a large effective step size and thus the proposed algorithm converges fast but results in a large misadjustment error. In case of \( \epsilon_a = \epsilon_e = K = 1 \), update equations of the proposed algorithm are similar to those of the normalized least mean square (NLMS) version of EE IIR filtering.
Table 1: Computational complexity of the MNEE vs. the proposed algorithm

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Multiplications</th>
<th>Additions</th>
</tr>
</thead>
<tbody>
<tr>
<td>MNEE [3]</td>
<td>$3L + 2M + 7$</td>
<td>$2L + 2M + 3$</td>
</tr>
<tr>
<td>proposed</td>
<td>$(K^2 + 2K + 2)L + (K^2 + 2K + 1)M + K^3 + 5K^2 + 4K + 5$</td>
<td>$(K^2 + 2K + 1)L + (K^2 + 2K + 1)M + K^2 + 2K^2 + 4K - 1$</td>
</tr>
</tbody>
</table>

Table 1 shows the number of multiplications and additions that are required in the evaluation of filter update at each iteration. Table 1 assumes that the cost of inverting a $K \times K$ matrix is $O(K^3)$ operations (multiplications and additions). In spite of increased computational complexity, the proposed algorithm shows a superior convergence rate than the MNEE in the following section.

3. Experimental Results

We illustrate the performance of the proposed algorithm by carrying out computer experiments in system identification. The MNEE is compared with the proposed algorithm. The monic normalization [3] is applied to the proposed algorithm for bias removal in $\hat{a}$. The unknown system, given by

$$H(z) = \frac{\alpha}{(1 - 0.9z^{-1})(1 + 0.81z^{-2})} (1 - 1.34z^{-1} + 0.9z^{-2})(1 + 0.75z^{-2} + 0.56z^{-3}),$$

(18)

which has four poles at $0.95 \pm 45^\circ$, $0.75 \pm 120^\circ$, and three zeros at $0.95 \pm 45^\circ$, $0.9$, was driven by a white, zero-mean, Gaussian random sequence having unit variance. $\alpha$ is the gain and is chosen such that $y(i)$ has unit power. For example, we use $\alpha = 0.5361$. The measurement noise sequence is white Gaussian with variance $\sigma^2$. The signal-to-noise ratio (SNR) is 30dB. The squared norms of the parameter errors, $\|a - \hat{a}\|^2$ and $\|b - \hat{b}\|^2$, are taken and averaged over 20 independent trials. The step sizes of MNEE are set to 0.0002. In order to achieve a similar misadjustment error with the MNEE, we set the step sizes and the regularization parameters of the proposed algorithm ($K = 1$) 0.009 and 20, respectively. In addition, we also set $\varepsilon_a = \varepsilon_b = 20$ and $\mu_a = \mu_b = 0.009$ in the proposed algorithm ($K = 2$).

Figure 2 and Figure 3 show $\|a - \hat{a}\|^2$ and $\|b - \hat{b}\|^2$, respectively. These results show that the convergence speed of the proposed algorithm is faster than that of the MNEE.

4. Conclusions

We have proposed a new adaptive EE IIR filtering scheme using vector EE with block regressors. The proposed algorithm is obtained from minimizing of the squared norm of

\[a \text{ posterior vector EE with respect to the changes of estimates. Consequently, we develop a block regressor EE adaptive IIR filtering algorithm. Experimental results demonstrate that the proposed algorithm improves convergence performance. Also numerical results of regularization parameters and the simplified algorithm are further issues.}\]

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References


