VECTOR EQUATION ERROR ADAPTIVE IIR FILTERING WITH BLOCK REGRESSORS

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ABSTRACT

This paper proposes a new equation error infinite impulse response (IIR) filtering algorithm using a vector equation error with block regressors. The squared norm of a posteriori vector equation error is introduced as a new cost function to be minimized with respect to the changes of filter estimates. New update equations with block regressors are derived as a consequence. Experimental results demonstrate that the proposed algorithm improves filter convergence performance.

Keywords: adaptive IIR filtering, vector equation error, block regressor.

1. INTRODUCTION

Adaptive IIR filtering methods are commonly classified into equation error (EE) and output error (OE) [1]. The EE approach for adaptive IIR filtering has such attractive features as a unimodal error surface, good convergence, and guaranteed system stability compared to the OE approach [1]. In spite of those advantages, the EE approach has not been widely used, since it may generate biased coefficient estimates in the presence of noise. Several algorithms are proposed to solve the bias problem [2]–[4].

Most existing EE IIR filtering has tried to minimize a squared scalar valued EE and uses non-block regressors. However, it is known that the block regressor gives birth to the improved convergence performance of adaptive filter in case of a finite impulse response (FIR) filtering [5], [6]. For example, the convergence performances using vector EE with block regressor such as the affine projection algorithm [5] and data-reusing algorithms [6] are better than those with non-block regressor. Motivated by this fact, we incorporate EE IIR filtering with block regressors.

In this paper, we introduce a new EE IIR filtering algorithm using vector EE with block regressors, which we will call the vector equation error adaptive IIR filtering. A new cost function to minimize the squared norm of a posterior vector EE with respect to the changes of filter estimates is defined. By minimizing the new cost function, update equations with block regressors are derived. Experimental results show that the convergence performance of the proposed algorithm is better than that of conventional EE IIR filtering.

2. VECTOR EQUATION ERROR ADAPTIVE IIR FILTERING

2.1. EE IIR Filtering

Let a zero-mean, discrete-time signal \( x(i) \) be the input to an unknown system \( H(z) \). Suppose that \( H(z) \) is represented by a rational function

\[
H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{m=0}^{M} b_m z^{-m}}{\sum_{l=0}^{L} a_l z^{-l}}, \tag{1}
\]

where \( a_l \) and \( b_m \) are the adaptive filter coefficients and \( z^{-l} \) denotes a delay of \( l \) samples. The output \( y(i) \) of the unknown system \( H(z) \) is represented by a difference equation.

Fig. 1: Adaptive EE IIR filter in system identification configuration.
Consider a posteriori vector EE
\[ e_p = D_i a_i - X_i b_i, \]
and a priori vector EE
\[ e_i = D_i a_{i-1} - X_i b_{i-1}, \]
where \( X_i \) and \( D_i \) are block regressors given by
\[
X_i^T = [x_i x_{i-1} \cdots x_{i-K+1}], \\
D_i^T = [d_i d_{i-1} \cdots d_{i-K+1}],
\]
and \( K \) is order of block regressors.

Using the newly defined vector EEs, we propose a new cost function
\[
J(i) = \|e_p\|^2 + \varepsilon_a \|\Delta a_{i-1}\|^2 + \varepsilon_b \|\Delta b_{i-1}\|^2 \\
= \|e_i + D_i \Delta a_{i-1} - X_i \Delta b_{i-1}\|^2 \\
+ \varepsilon_a \|\Delta a_{i-1}\|^2 + \varepsilon_b \|\Delta b_{i-1}\|^2,
\]
where the changes of filter estimates, \( \Delta a_{i-1} \) and \( \Delta b_{i-1} \), are given by
\[
\Delta a_{i-1} = a_i - a_{i-1}, \\
\Delta b_{i-1} = b_i - b_{i-1},
\]
and \( \varepsilon_a \) and \( \varepsilon_b \) are positive regularization parameters of \( \Delta a_{i-1} \) and \( \Delta b_{i-1} \), respectively.

Taking derivatives of \( J(i) \) with respect to \( \Delta a_{i-1} \) and \( \Delta b_{i-1} \) results in
\[
\frac{\partial J(i)}{\partial \Delta a_{i-1}} = 2D_i^T (e_i + D_i \Delta a_{i-1} - X_i \Delta b_{i-1}) \\
+ 2\varepsilon_a \Delta a_{i-1},
\]
and
\[
\frac{\partial J(i)}{\partial \Delta b_{i-1}} = -2X_i^T (e_i + D_i \Delta a_{i-1} - X_i \Delta b_{i-1}) \\
+ 2\varepsilon_b \Delta b_{i-1}.
\]
Setting (11) and (12) equal to zero, we get
\[
\begin{bmatrix}
D_i^T D_i + \varepsilon_a I_{L+1} & D_i^T X_i \\
X_i^T D_i & X_i^T X_i + \varepsilon_b I_{M+1}
\end{bmatrix}
\begin{bmatrix}
-\Delta a_{i-1} \\
-\Delta b_{i-1}
\end{bmatrix}
= \begin{bmatrix}
D_i^T \\
X_i^T
\end{bmatrix} e_i,
\]
where \( D_i^T D_i + \varepsilon_a I_{L+1} \) and \( X_i^T X_i + \varepsilon_b I_{M+1} \) is positive definite and \( I_K \) is \( K \times K \) identity matrix. Rewriting (13) with respect to \( \Delta a_{i-1} \) and \( \Delta b_{i-1} \) yields
\[
\begin{bmatrix}
-\Delta a_{i-1} \\
-\Delta b_{i-1}
\end{bmatrix}
= \begin{bmatrix}
D_i^T D_i + \varepsilon_a I_{L+1} & D_i^T X_i \\
X_i^T D_i & X_i^T X_i + \varepsilon_b I_{M+1}
\end{bmatrix}
^{-1}
\begin{bmatrix}
D_i^T \\
X_i^T
\end{bmatrix} e_i
= \left( \begin{bmatrix}
\varepsilon_a I_{L+1} & 0 \\
0 & \varepsilon_b I_{M+1}
\end{bmatrix}
+ \begin{bmatrix}
D_i^T X_i \\
X_i^T X_i + \varepsilon_b I_{M+1}
\end{bmatrix}
\right)^{-1}
\begin{bmatrix}
D_i^T \\
X_i^T
\end{bmatrix} e_i.
\]
By matrix inversion lemma [7],

\[
\begin{bmatrix}
\epsilon a I_{L+1} & 0 \\
0 & \epsilon b I_{M+1}
\end{bmatrix}
+ \begin{bmatrix}
D^T \\
X_i^T
\end{bmatrix}
\begin{bmatrix}
D_i & X_i
\end{bmatrix}^{-1}
\begin{bmatrix}
D^T \\
X_i^T
\end{bmatrix}
= \frac{1}{\epsilon a} I_{L+1} + \frac{1}{\epsilon b} I_{M+1}
\begin{bmatrix}
I_K + \frac{1}{\epsilon a} D_i D_i^T + \frac{1}{\epsilon b} X_i X_i^T
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
D^T \\
X_i^T
\end{bmatrix}.
\]

Substituting (15) into (14) and using (9) and (10) produce the following update equations:

\[
\hat{a}_i = \hat{a}_{i-1} - \frac{\mu_a}{\epsilon a} D_i^T \left( I_K + \frac{1}{\epsilon a} D_i D_i^T + \frac{1}{\epsilon b} X_i X_i^T \right)^{-1} e_i,
\]

(16)

\[
\hat{b}_i = \hat{b}_{i-1} + \frac{\mu_b}{\epsilon b} X_i^T \left( I_K + \frac{1}{\epsilon a} D_i D_i^T + \frac{1}{\epsilon b} X_i X_i^T \right)^{-1} e_i,
\]

(17)

where \(\mu_a\) and \(\mu_b\) are the step sizes which control the convergence speed and the misadjustment error after convergence.

From (16) and (17), we know that \(\epsilon_a\) and \(\epsilon_b\) take part in the convergence performance with \(\mu_a\) and \(\mu_b\). In case of \(\epsilon_a = \epsilon_b\), a large regularization parameter will ensure a small effective step size and thus the proposed algorithm results in a small misadjustment error in steady state, but converges slowly. On the other hand, a small regularization parameter will yield a large effective step size and thus the proposed algorithm converges fast but results in a large misadjustment error. In case of \(\epsilon_a = \epsilon_b\) and \(K = 1\), update equations of the proposed algorithm are similar to those of the normalized least mean square (NLMS) version of EE IIR filtering.

Table 1 shows the number of multiplications and additions that are required in the evaluation of filter update at each iteration. Table 1 assumes that the cost of inverting a \(K \times K\) matrix is \(O(K^3)\) operations (multiplications and additions). In spite of increased computational complexity, the proposed algorithm shows a superior convergence rate than the MNEE in the following section.

### 3. EXPERIMENTAL RESULTS

We illustrate the performance of the proposed algorithm by carrying out computer experiments in system identification. The MNEE is compared with the proposed algorithm. The monic normalization [3] is ap-

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**Table 1: Computational complexity**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>multiplications</th>
<th>additions</th>
</tr>
</thead>
<tbody>
<tr>
<td>MNEE [3]</td>
<td>(3L + 2M + 7)</td>
<td>(2L + 2M + 3)</td>
</tr>
<tr>
<td>proposed</td>
<td>((K^2 + 2K + 2)L) + ((K^2 + 2K + 1)M) + (K^3 + 5K^2 + 4K) + 5</td>
<td>((K^2 + 2K + 1)L) + ((K^2 + 2K + 1)M) + (K^3 + 2K^2 + 4K) - 1</td>
</tr>
</tbody>
</table>

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**Fig. 2:** Performances of the proposed algorithm and the MNEE of \(\hat{a}_i\), \(\|a - \hat{a}_i\|^2\). (a) MNEE, (b) proposed \((K = 1)\), (c) proposed \((K = 2)\), (d) proposed \((K = 4)\).

**Fig. 3:** Performances of the proposed algorithm and the MNEE of \(\hat{b}_i\), \(\|b - \hat{b}_i\|^2\). (a) MNEE, (b) proposed \((K = 1)\), (c) proposed \((K = 2)\), (d) proposed \((K = 4)\).
plied to the proposed algorithm for bias removal in $\hat{a}_i$. The unknown system, given by

$$H(z) = \frac{\alpha}{(1 - 0.9z^{-1})(1 + 0.81z^{-2})} \frac{(1 - 1.34z^{-1} + 0.9z^{-2})(1 + 0.75z^{-1} + 0.56z^{-2})}{(1 - 1.34z^{-1} + 0.9z^{-2})(1 + 0.75z^{-1} + 0.56z^{-2})} \tag{18}$$

which has four poles at $0.95\angle \pm 45^\circ$, $0.75\angle \pm 120^\circ$, and three zeros at $0.9\angle \pm 90^\circ$, $0.9$, was driven by a white, zero-mean, Gaussian random sequence having unit variance. $\alpha$ is the gain and is chosen such that $y(i)$ has unit power. For example, we use $\alpha = 0.5361$. The measurement noise sequence is white Gaussian with variance $\sigma^2_v$. The signal-to-noise ratio (SNR) is 30dB. The squared norms of the parameter errors, $\|a - \hat{a}_i\|^2$ and $\|b - \hat{b}_i\|^2$, are taken and averaged over 20 independent trials. The step sizes of MNEE are set to 0.0002. In order to achieve a similar misadjustment error with the MNEE, we set the step sizes and the regularization parameters of the proposed algorithm ($K = 1$) $0.009$ and $20$, respectively. In addition, we also set $\epsilon_a = \epsilon_b = 20$ and $\mu_a = \mu_b = 0.009$ in the proposed algorithm ($K \geq 2$).

Fig. 2 and Fig. 3 show $\|a - \hat{a}_i\|^2$ and $\|b - \hat{b}_i\|^2$, respectively. These results show that the convergence speed of the proposed algorithm is faster than that of the MNEE.

4. CONCLUSIONS

In this paper, a new adaptive EE IIR filtering scheme using vector EE with block regressors has been proposed. A new cost function, the squared norm of a posteriori vector EE, to be minimized with respect to the changes of filter estimates is introduced. Consequently, we derived new updated equations with block regressor based EE adaptive IIR filtering algorithm. Experimental results demonstrate that the proposed algorithm possesses the improved convergence performance. Also, numerical quantity of regularization parameters and the simplified algorithm remain as further issues.

5. ACKNOWLEDGEMENT

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6. REFERENCES


