Improved $L_\infty$-norm Adaptive Filtering Algorithm with Sparse Updates

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Abstract: This paper provides an improved low-complexity minimum $L_\infty$-norm adaptive filtering algorithm with sparse updates. The proposed algorithm overcomes initial adversity in the convergence performance of the conventional low-complexity minimum $L_\infty$-norm adaptive filtering algorithm with sparse updates due to a large step-size. Experimental results show that the proposed algorithm has better convergence performance than the conventional algorithm.

Keywords: Adaptive filtering, normalized least mean square (NLMS), $L_\infty$-norm, sparse updates

I. Introduction

In the adaptive filtering algorithms, we have the choice of algorithms ranging from the simple least-mean square (LMS) algorithm to the more complex recursive least squares (RLS) algorithm with tradeoffs between the computational complexity and the convergence speed.

Among of them, the normalized least-mean square (NLMS) algorithm is widely used due to its low computational complexity and simplicity. However, the usefulness of the NLMS algorithm may be diminished for a system designed with a large number of coefficients due to increased complexity [1], [2]. So as to reduce the complexity, the normalized sign LMS (NSLMS) algorithm based on minimum $L_\infty$-norm has been developed [3], [4]. The complexity of the NSLMS algorithm is reduced by updating the filter coefficients in a quantized form. Despite this conspicuous advantage, the NSLMS algorithm still is not sufficient to be used for the large number of filter coefficients. As another approach to reduce computational complexity, the reduction of updates of the filter coefficients, which is sparse in time, can reduce the average computational complexity. These algorithms can be derived within the framework of set-membership filtering (SMF), which employs a bounded error constraint on the filter output [5].

In [6], the authors proposed the low-complexity minimum $L_\infty$-norm adaptive filtering algorithm with sparse updates, called NSLMS with sparse updates (NSLMS-SU), which has lower steady-state error than the conventional $L_\infty$-norm adaptive filtering algorithm [4] with greatly reduced overall computational complexity. However, the related issues of the slow initial convergence of the NSLMS-SU have not been discussed yet.

In this paper, we propose an improved NSLMS-SU (INSLMS-SU) algorithm which enhances the initial convergence performance through the geometric interpretation.

Throughout this paper, the following norms are adopted:

$$|| \cdot || = \text{Euclidean norm (} - \text{norm)}$$

This paper is organized as follows. Section 2 introduces the conventional NSLMS-SU algorithm. In addition, we propose a novel updating strategy on which the INSLMS-SU is derived based. Section 3 contains experimental results which illustrate the convergence performance of the proposed algorithm. Finally, conclusions are presented in Section 5.

II. Improved NSLMS-SU (INSLMS-SU) Algorithm

Consider a desired signal $d(i)$ which is obtained from the linear model

$$d(i) = w^T u + \nu(i)$$

where $w$ is an unknown vector to be identified with an adaptive filter $u$, $\nu(i)$ is measurement noise, $u$ denotes the $N \times 1$ input vector,

$$u = [u(i) u(i-1) \cdots u(i-N+1)]^T.$$ 

Then, the estimation error is given by

$$e(i) = d(i) - w^T u.$$ 

1. Conventional NSLMS-SU

The conventional NSLMS-SU algorithm would be derived to solve the constrained minimization problem given by

$$\min \|w_{i+1} - w_i\|,$$

subject to $\|d(i) - w_{i+1}^T u\| \leq \gamma$. 

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As in the error control procedure described in [4], we can obtain the filter coefficient vector \( \mathbf{w}_{i+1} \). Thus the update equation of the conventional NSLMS-SU algorithm is given by

\[
\mathbf{w}_{i+1} = \mathbf{w}_i + \frac{\mu(i) e(i)}{||\mathbf{u}||} \text{sign}(\mathbf{u}) \tag{5}
\]

where

\[
||\mathbf{u}|| = \sum_{n=0}^{N-1} |u(i-n)| \tag{6}
\]

and

\[
\text{sign}(\mathbf{u}) = [s_0(i) s_1(i) \cdots s_{N-1}(i)]^T
\]

\[
s_k(i) = \begin{cases} +1, & \text{where } u(i-n+1) \geq 0 \\ -1, & \text{where } u(i-n+1) < 0 \end{cases}
\]

\[
n = 1, \ldots, N.
\]

It is shown [6] that the step-size is determined so as to satisfy the constraint in (4) as follows:

\[
\mu(i) = \begin{cases} 1 - \gamma |e(i)|, & \text{where } |e(i)| > \gamma \\ 0, & \text{otherwise.} \end{cases}
\]

2. Proposed INSLMS-SU

In [6], the NSLMS-SU algorithm is appropriate for the system with the large number of filter coefficients, since it profits from the sparse updates related to the set-membership framework reducing the average computational complexity, as well as from the reduced computational complexity with the \( L_\infty \)-norm adaptive filtering. In addition, it is demonstrated that the NSLMS-SU algorithm outperforms the NSLMS algorithm in terms of steady-state error. However, the NSLMS-SU algorithm suffers from the degraded initial convergence because the NSLMS algorithm has poor convergence performance when the step-size \( \mu \) is very close to 1. To overcome this problem, we provide following update strategies.

Let \( \Theta \) shown in Fig. 1 denote the angle between the direction of \( \text{sign}(\mathbf{u}) \) and \( \mathbf{u} \), then \( \cos \Theta \) is given by

\[
\cos \Theta = \frac{||\mathbf{w}_i - \mathbf{w}_{SM-NLMS,i+1}||}{||\mathbf{w}_i - \mathbf{w}_{i+1}||} \tag{9}
\]

where the \( \mathbf{w}_{SM-NLMS,i+1} \) is obtained by

\[
\mathbf{w}_{SM-NLMS,i+1} = \mathbf{w}_i + \frac{\mu(i) e(i)}{||\mathbf{u}||} \tag{10}
\]

in the SM-NLMS algorithm [5]. We define \( \theta_\perp \) as the angle \( \theta \) in the case of \( (\mathbf{w}_{NLMS,i+1} - \mathbf{w}_{i+1}) \perp (\mathbf{w}_i - \mathbf{w}_{i+1}) \) where the \( \mathbf{w}_{NLMS,i+1} \) is obtained by

\[
\mathbf{w}_{NLMS,i+1} = \mathbf{w}_i + \frac{e(i)}{||\mathbf{u}||} \tag{11}
\]

in the NLMS algorithm with unit step-size [2].

It is acceptable that the convergence rate of the updating equation improves when \( \mathbf{w}_i \) is updated to the closest point to \( \mathbf{w}_{NLMS,i+1} \) per every iteration. Thus, if the inequality, \( \theta > \theta_\perp \), is satisfied, let us increase the error bound \( \gamma \) temporarily at the iteration \( i \) to \( \gamma_i \) so as to be

\[
(\mathbf{w}_{NLMS,i+1} - \mathbf{w}_{i+1}) \perp (\mathbf{w}_i - \mathbf{w}_{i+1}) \tag{12}
\]

as illustrated in Fig. 1. Then, \( \cos \theta \) can be also represented by

\[
\cos \theta = \frac{||\mathbf{w}_i - \mathbf{w}_{i+1}||}{||\mathbf{w}_i - \mathbf{w}_{NLMS,i+1}||} \tag{13}
\]

From (10) and (11),

\[
||\mathbf{w}_i - \mathbf{w}_{SM-NLMS,i+1}|| = ||\mu(i) e(i) u||/||\mathbf{u}||
\]

and

\[
||\mathbf{w}_i - \mathbf{w}_{NLMS,i+1}|| = ||e(i) u||/||\mathbf{u}||.
\]

In addition, we can calculate \( ||\mathbf{w}_i - \mathbf{w}_{i+1}|| \) from (5) such that

\[
||\mathbf{w}_i - \mathbf{w}_{i+1}|| = \sqrt{N} ||\mu(i) e(i) / ||\mathbf{u}||. \tag{14}
\]

The update equation for \( \mathbf{w}_{i+1} \) has a modified step-size \( \mu'(i) \) due to expended \( \gamma_i \), as follows:

\[
\mathbf{w}'_{i+1} = \mathbf{w}_i + \frac{\mu'(i) e(i)}{||\mathbf{u}||} \text{sign}(\mathbf{u}) \tag{15}
\]

Hence, \( ||\mathbf{w}_i - \mathbf{w}'_{i+1}|| = \sqrt{N} ||\mu'(i) e(i) / ||\mathbf{u}||. \) Consequently, the step-size \( \mu'(i) \) can be calculated by the equality of (9) and (13), which is given by

\[
\mu'(i) = \frac{||\mathbf{u}||}{\sqrt{N} ||\mathbf{u}||}. \tag{16}
\]

Therefore, the proposed update equation is given by

\[
\mathbf{w}_{i+1} = \mathbf{w}_i + \frac{\mu(i) e(i)}{||\mathbf{u}||} \text{sign}(\mathbf{u}) \tag{17}
\]

where

\[
\mu(i) = \begin{cases} 1 - \gamma |e(i)|, & \text{where } |e(i)| > \gamma \\ 0, & \text{otherwise.} \end{cases}
\]

However, if \( \theta > \theta_\perp \), i.e., \( ||\mathbf{u}|| < \mu(i) N ||\mathbf{u}|| \), then \( \mu(i) \) is substituted with \( \mu'(i) \).
Ⅲ. Experimental Results

We illustrate the performance of the proposed algorithm by carrying out computer simulation in a channel identification scenario. The order of unknown channel shown in Fig. 2 is \( N=128 \). The adaptive filter and the unknown channel are assumed to have the same number of taps. The input signal is randomly generated from a stationary, white Gaussian process with zero mean and unit variance and is also obtained by filtering the generated signal through the system whose transfer function is given by

\[
G(z) = \frac{1}{1 - 0.9z^{-1}}.
\]  

(19)

The signal-to-noise ratio (SNR) is set to 30 dB which is calculated by \( \text{SNR} = 10 \log_{10}(E[y^2(i)]/E[v^2(i)]) \), where \( y(i) = w[i]u[i] \). The mean square error (MSE), \( E[e(i)^2] \), is taken and averaged over 5000 independent trials. We set the step-size of the NSLMS algorithms as 0.6.

In Fig. 3, it is shown the MSE curves of the NSLMS, the NSLMS-SU, and the proposed INSLMS-SU algorithm [Input: white Gaussian].

In Fig. 4, we see that the NSLMS algorithm does not converge with \( \mu = 1.0 \) or 0.9, and has poor convergence performance with \( \mu = 0.8 \) for the above colored input signals. For that reason, since the conventional NSLMS-SU has initially a large step-size which is close to 1, the algorithm may diverge, as shown in Fig. 5. However, the proposed INSLMS-SU algorithm converges well and outperforms the NSLMS algorithm with reduced overall computational complexity.

Gaussian input signals. The error bound \( \gamma \) is set to \( \sqrt{3\sigma^2} \), where \( \sigma^2 \) is the variance of the measurement noise. As can be seen, the proposed method shows improved convergence performance than the conventional NSLMS-SU and outperforms the NSLMS algorithm with reduced overall computational complexity.

In Figs. 4 and 5, we choose \( \gamma = \sqrt{3\sigma^2} \) and different input signals generated by \( G(z) \). In Fig. 4, we see that the NSLMS algorithm does not converge with \( \mu = 1.0 \) or 0.9, and has poor convergence performance with \( \mu = 0.8 \) for the above colored input signals. For that reason, since the conventional NSLMS-SU has initially a large step-size which is close to 1, the algorithm may diverge, as shown in Fig. 5. However, the proposed INSLMS-SU algorithm converges well and outperforms the NSLMS algorithm with reduced overall computational complexity.
IV. Conclusion

We have proposed a modified version of the conventional low complexity minimum $L_\infty$-norm adaptive filtering algorithm with sparse updates. The proposed algorithm has better convergence performance than the NSLMS algorithm having the reduced complexity and is stable compared to the conventional NSLMS algorithm with sparse updates. The related issues of the convergence and the stability of the proposed algorithm are not thoroughly covered in this paper, which is left for a future work.

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